Effect of Heat Generation on Magnetohydrodynamic(MHD) Free Convection Flow with Conduction

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Abstract

This paper reports a numerical study of the magnetohydrodynamic(MHD) free convection heat transfer taking into account heat generation in presence of heat conduction. The developed governing equations and the associated boundary conditions for this analysis are transferred to dimensionless forms using a suitable transformation. The transformed non-dimensional governing equations are then solved using the implicit finite difference method with Keller box-scheme. Numerical outcomes are found for different values of the Magnetic parameter and heat generation parameter in terms of velocity profiles, temperature distributions, skin friction coefficient and heat transfer rate. All these results are shown graphically with a complete discussion.

Key words: Free convection, MHD, Heat generation, Conduction, Horizontal cylinder.

Introduction

The conjugate heat transfer process (CHT) formed by the interaction between the conduction inside the solid and the convection flow along the solid surface has significant importance in many practical applications. In fact, conduction within the tube wall is significantly influenced by the convection in the surrounding fluid. On the other hand, many practical heat transfer applications involve the conversion of some form of energy into thermal energy in the medium. Such mediums are said to involve internal heat generation. The study of heat generation in moving fluids is important specially when it deals with chemical reaction. Possible heat generation effects may modify temperature distribution and, therefore, particle deposition rate. Consequently, the conduction and heat generation in the solid body and the convection in the fluid should have to be determined simultaneously.

The natural convection flow of an incompressible and viscous fluid from horizontal cylinder was studied by several research groups [1-4]. In these studies, wall conduction resistance for the convective heat transfer and internal heat generation was ignored.

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Gdalevich and Fertman [5] studied the conjugate problems of natural convection. Miyamoto et al. [6] analysed the effects of axial heat conduction in a vertical flat plate on free convection heat transfer. Miyamoto observed that a mixed-problem study of the natural convection has to be performed for an accurate analysis of the thermo-fluid-dynamic (TFD) field, if the convective heat transfer depends strongly on the thermal boundary conditions. Pozzi et al. [7] investigated the entire TFD field resulting from the coupling of natural convection along and conduction inside a heated flat plate by means of two expansions, regular series and asymptotic expansions. Moreover, Kimura and Pop [8] analysed conjugate natural convection from a horizontal circular cylinder.

The heat transfer in a laminar boundary layer flow of a viscous fluid over a linearly stretching continuous surface with viscous dissipation/frictional heating and internal heat generation was analyzed by Vajravelu and Hadjinicolaou [9]. They considered the volumetric rate of heat generation, q''' [W/m^3], as $q''' = Q_0 \left(T_f - T_\infty\right), \; for \; T_f \geq T_\infty$ and $q''' = 0, \; for \; T_f < T_\infty$, where Q_0 is the heat generation, constant. The above relation is valid for the state of some exothermic processes having T_∞ as onset temperature. In the present study we considered the above term as heat generation.

Magnetohydrodynamic (MHD) flow and heat transfer process are now an important research area due to its potential application in engineering and industrial fields. A considerable amount of research has been done in this field. Wilks et al. [10] studied MHD free convection about a semi-infinite vertical plate in a strong cross field. Takhar and Soundalgekar [11] investigated dissipation effects on MHD free convection flow past a semi-infinite vertical plate. Hossain [12] studied viscous and Joule heating effects on MHD free convection flow with variable plate temperature. Aldoss et al. [13] analysed MHD mixed convection from a horizontal circular cylinder. El-Amin [14] found out the combined effect of viscous dissipation and Joule heating on MHD forced convection over a non-isothermal horizontal circular cylinder embedded in a fluid saturated porous medium.

To our best knowledge, the effect of MHD free convection considering internal heat generation of the surrounding fluid in presence of heat conduction from an isothermal circular cylinder has not been studied yet. In this paper, the effect of heat generation on MHD free convection flow from an isothermal horizontal circular cylinder in presence of heat conduction is considered.

Mathematical Analysis

Let us consider a steady natural convection flow of a viscous incompressible and electrically conducting fluid from an isothermal horizontal circular cylinder of radius **a** placed in a fluid

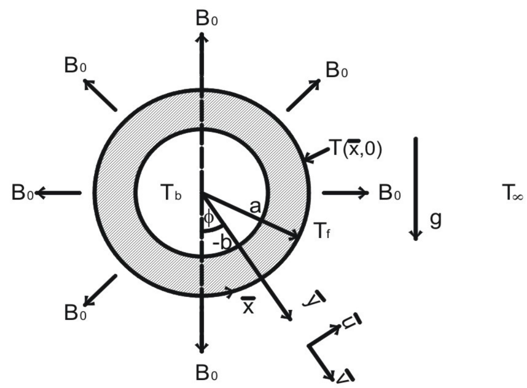


Fig. 1: Physical Model and coordinate system

of uniform temperature T_∞. The cylinder has a heated core region of temperature T_b and the normal distance from inner surface on the outer surface is b with $T_b > T_\infty$. A uniform magnetic field having strength B_0 is acting normal to the cylinder surface. The x-axis is taken along the circumference of the cylinder measured from the lower stagnation point and the y-axis is taken normal to the surface. It is assumed that the fluid properties are constant and the induced magnetic field is ignored. Under the balance laws of mass, momentum and energy and with the help of Boussinesq approximation for the body force term in the momentum equation, the equations governing this boundary-layer natural convection flow can be written as:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta\left(T_f - T_\infty\right)\sin\left(\frac{\bar{x}}{a}\right) - \frac{\sigma B_0^{\,2}\bar{u}}{\rho} \tag{2}$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa_f}{\rho c_p}\frac{\partial^2 T_f}{\partial \bar{y}^2} + \frac{Q_0}{\rho c_p}\left(T_f - T_\infty\right) \tag{3}$$

The appropriate boundary conditions for the problem

$$\bar{u} = \bar{v} = 0,\ T_f = T(\bar{x},0),\ \ \frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa_s}{b\kappa_f}\left(T_f - T_b\right)\ on\ \bar{y} = 0,\ x > 0 \tag{4}$$

$$\bar{u} \to 0,\ T_f \to T_\infty\ as\ \bar{y} \to \infty,\ \bar{x} > 0$$

The governing equations and the boundary conditions (1)-(4) can be made non-dimensional, using the Grashof number $Gr=[g\beta a^3(T_b-T_\infty)]/\nu^2$ which is assumed large and the following non-dimensional variables:

$$x=\frac{\bar{x}}{a},\ y=\frac{\bar{y}}{a}Gr^{1/4},\ u=\frac{\bar{u}a}{\nu}Gr^{-1/2},\ v=\frac{\bar{v}a}{\nu}Gr^{-1/4},\ \theta=\frac{T_f-T_\infty}{T_b-T_\infty} \tag{5}$$

where θ is the dimensionless temperature. The non dimensional form of the equations (1)-(3) are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+Mu=\frac{\partial^2 u}{\partial y^2}+\theta\sin x \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+Q\theta \tag{8}$$

where $M=\left(\sigma a^2 B_0{}^2\right)/\left(\nu\rho Gr^{1/2}\right)$ is the magnetic parameter, $Q=\left(Q_0 a^2\right)/\left(\nu\rho c_p Gr^{1/2}\right)$ is the heat generation parameter and $Pr=\left(\mu c_p\right)/\left(\kappa_f\right)$ is the Prandtl number.

The boundary condition (4) can be written as it is in the following dimensionless form:

$$u=v=0,\ \theta-1=p\frac{\partial\theta}{\partial y},\ on\ y=0,\ x>0 \tag{9}$$

$$u\to 0,\ \theta\to 0\ as\ y\to\infty,\ x>0$$

where $p=\left(b\kappa_f Gr^{1/4}\right)/\left(a\kappa_s\right)$ is the conjugate conduction parameter. The present problem is governed by the magnitude of p. The values of p depends on b/a, κ_f/κ_s and Gr. These ratios b/a and κ_f/κ_s are less than one where as Gr is large for free convection. Therefore the value of p is equal to zero (b =0) or greater than zero. In the present investigation we considered p=1 to ensure the presence of heat conduction.

To solve equation (6)-(8), subject to the boundary condition (9), we assume following transformations:

$$\psi=x\,f\left(x,y\right),\ \theta=\theta(x,y) \tag{10}$$

where ψ is the stream function usually defined as

$$u=\partial\psi/\partial y,\ v=-\partial\psi/\partial x. \tag{11}$$

Substituting (11) into the equations (6)-(9), new forms of the equations (7) and (8) are:

$$f'''+ff''-f'^2-Mf'+\theta\frac{\sin x}{x}=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \tag{12}$$

$$\frac{1}{\Pr}\theta'' + f\theta' + Q\theta = x\left(f'\frac{\partial\theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \quad (13)$$

In the above equations primes denote differentiation with respect to *y*. The corresponding boundary conditions take the following form

$$f = f' = 0,\ \theta - 1 = \frac{\partial\theta}{\partial y}\ at\ \ y = 0,\ x > 0$$
$$f' \to 0,\ \theta \to 0\ as\ \ y \to \infty,\ x > 0 \quad (14)$$

If the value of *p* is taken to be zero, the boundary condition in equation (9) will change to

$$u = v = 0,\ \theta = 1,\ \ on\ \ y = 0,\ x > 0$$
$$u \to 0,\ \theta \to 0\ as\ \ y \to \infty,\ x > 0 \quad (15)$$

The present problem with M=0.0 in equation (12) and Q=0.0 in equation (13) and the boundary conditions described in equation (15) are the problems which were introduced by Merkin [1].

The principle physical quantities, the shearing stress and the rate of heat transfer in terms of skin friction coefficient C_f and Nusselt number *Nu* respectively can be written as

$$C_f\,Gr^{1/4} = x\,f''(x,0)\,,\ Nu\,Gr^{-1/4} = -\theta(x,0) \quad (16)$$

The results of the velocity profiles and the temperature distributions can be calculated by the following relations respectively:

$$u = f'(x, y)\,,\ \theta = \theta(x, y) \quad (17)$$

Method of Solution

Equations (12) and (13) are solved numerically based on the boundary conditions as described in equation (14) and (15) using one of the most efficient and accurate methods known as implicit finite difference method [16] with Keller box scheme [15].

Results and Discussion

The main objective of the present work is to analyze the flow of the fluid and the heat transfer processes due to the heat generation on MHD conjugate free convection flow from an isothermal horizontal circular cylinder. The value of the Prandtl number Pr = 1.0 is considered for the simulation that corresponds to steam.

A comparison of the local Nusselt number and the local skin friction factor obtained in the present work with M = 0.0, Q = 0.0, p = 0.0 and Pr = 1.0 and obtained by Merkin [1] and Nazar et al. [14] have been shown in Table 1. There is an excellent agreement among these three results. The velocity and the temperature distributions at $x = \pi/6$ against *y* for different values of heat generation parameter Q, with M = 0.2 are shown in fig.2(a)-(b). It is observed from these two figures that both the velocity and the temperature increase for the increasing values of heat generation parameter. It is expected because heat generation in the fluid increases the temperature within the boundary layer which accelerates the convection as well as increases the flow within the boundary layer. Fig. 3(a)-(b) represents the local skin friction coefficients and the local Nusselt number against *x* for different values of heat generation parameter Q. The fluid flow accelerates with increasing Q accordingly, it enhances the local skin friction coefficient as observed in fig.3(a). From fig. 3(b) it is observed that the local Nusselt number decreases for large values of heat generation parameter Q.

Table 1: Numerical values of $-\theta'(x,0)$ and $x f''(x,0)$ for different values of x while Pr = 1.0, M = 0.0, Q = 0.0, and p = 0.0.

x	$Nu\,Gr^{-1/4} = -\theta'(x,0)$			$C_f\,Gr^{1/4} = x f''(x,0)$		
	Merkin [1]	Nazar et al. [15]	Present	Merkin [1]	Nazar et al. [15]	Present
0.0	0.4214	0.4214	0.4216	0.0000	0.0000	0.0000
$\pi/6$	0.4161	0.4161	0.4163	0.4151	0.4148	0.4139
$\pi/3$	0.4007	0.4005	0.4006	0.7558	0.7542	0.7528
$\pi/2$	0.3745	0.3741	0.3741	0.9579	0.9545	0.9526
$2\pi/3$	0.3364	0.3355	0.3355	0.9756	0.9698	0.9678
$5\pi/6$	0.2825	0.2811	0.2811	0.7822	0.7740	0.7718
π	0.1945	0.1916	0.1912	0.3391	0.3265	0.3239

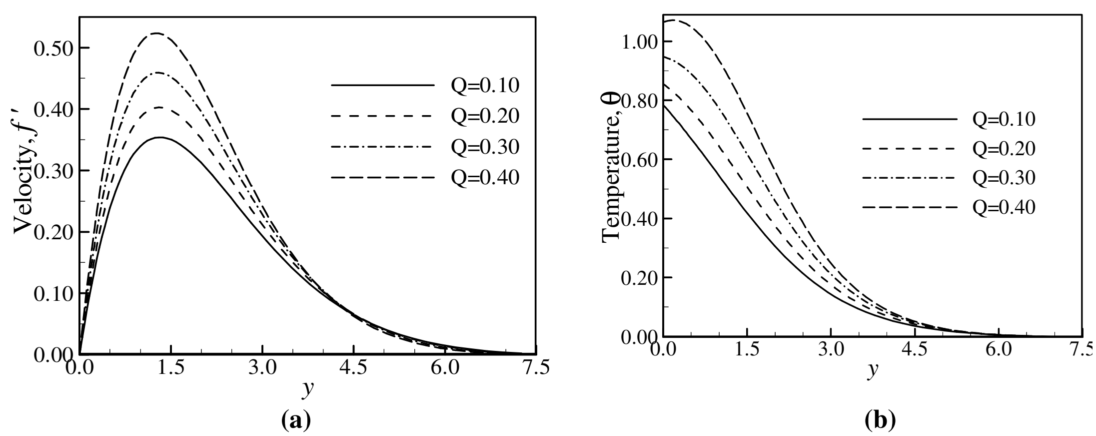
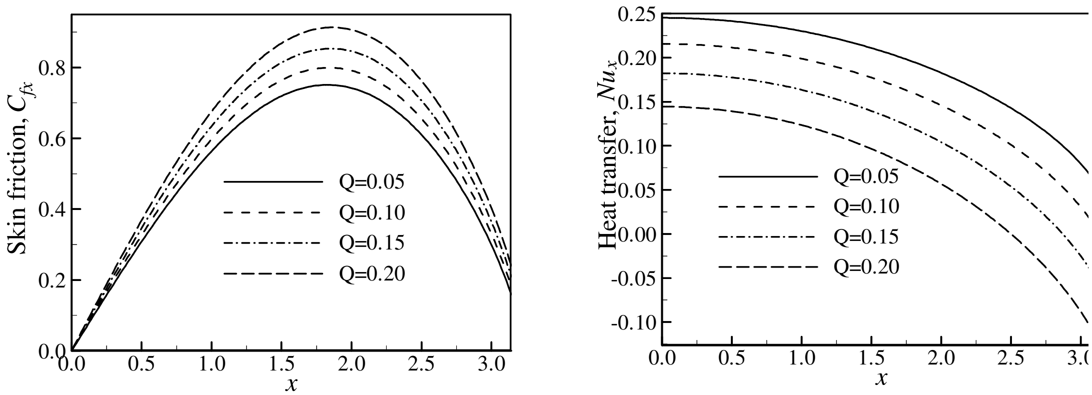


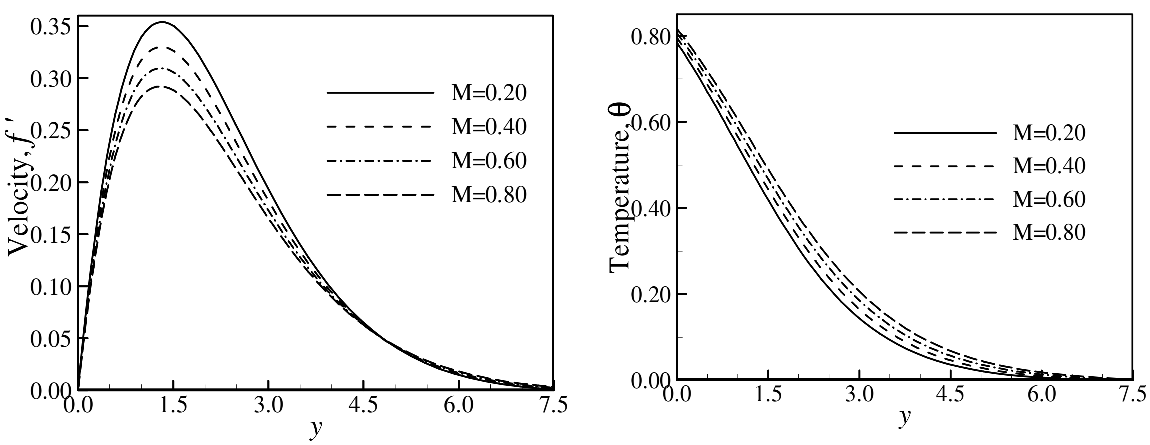
Fig.2: (a) Variation of velocity profiles and (b) variation of temperature profiles against *y* for varying of heat generation parameter Q with Pr = 1.0, M =0.2 and p = 1.0.



(a) **(b)**

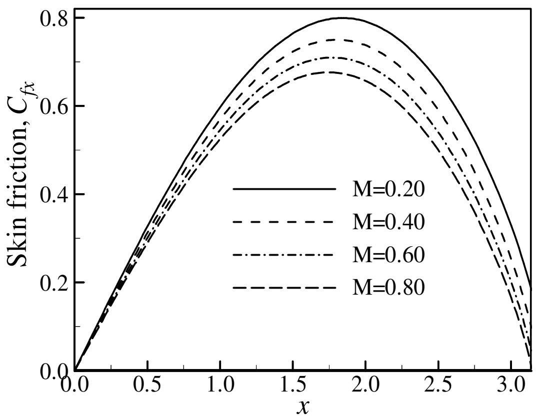
Fig.3: (a) Variation of the local skin friction coefficients and (b) variation of local Nusselt number against for varying of heat generation parameter Q with Pr = 1.0, M =0.2 and p = 1.0.

The magnetic field opposes the fluid flow. As a result the velocity decreases with the increasing M as shown in fig. 4(a) and the peak velocity moves towards the cylinder surface. Consequently, the separation of the boundary layer occurs earlier and the momentum boundary layer becomes thicker. From Fig. 4(b) it can be observed that increasing value of the magnetic parameter increases the temperature in the boundary layer for a particular value of y. Thus, the magnetic parameter increases the thickness of the thermal boundary layer.

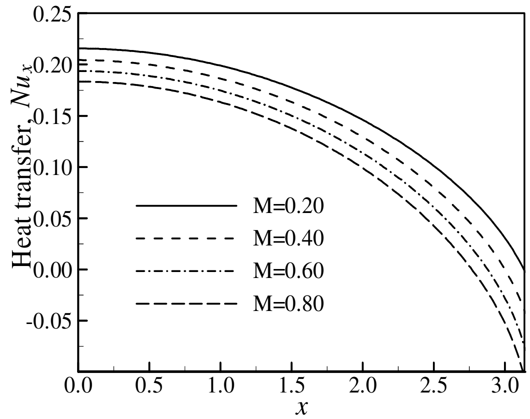


(a) **(b)**

Fig.4: (a) Variation of velocity profiles and (b) variation of temperature profiles against y for varying of magnetic parameter M with Pr = 1.0, Q = 0.1 and p = 1.0.



(a)



(b)

Fig.5: (a) Variation of the local skin friction coefficients and (b) variation of the local Nusselt number against x for varying of magnetic parameter M with $Pr = 1.0$, Q = 0.1 and $p = 1.0$.

Temperature at the interface also varies with different M since the conduction is considered within cylinder. The variation of the local skin friction coefficient and local Nusselt number with $Q = 0.1$ and p=1.0 for different values of M at different positions are illustrated in Fig.5(a) and Fig.5(b). The Magnetic force opposes the flow, as mentioned earlier, and reduces the shear stress at the wall as illustrated in Fig.5(a). Moreover, the heat transfer rate also decreases as revealed in Fig.5(b). From figs. 2(b) and 4(b), it is observed that, temperature profiles begin with different temperatures at the interface for different values of heat generation parameter and magnetic parameter respectively. Clearly, the presence of heat conduction influences temperature within the boundary layer.

Conclusion

The effects of the heat generation parameter Q and the magnetic parameter M with Prandtl number $Pr = 1.0$ in presence of heat conduction are analysed. The velocity within the boundary layer increases for increasing heat generation parameter Q whereas it decreases for increasing magnetic parameter M. Temperature within the boundary layer increases for increasing heat generation parameter Q and magnetic parameter M. The skin friction increases and heat transfer decreases for increasing heat generation parameter Q. On the other hand both the skin friction and the heat transfer rate decrease for the increasing value of magnetic parameter M. Finally, it is found that heat conduction has a significant influence in the temperature within the boundary layer; consequently, it may influence heat transfer, velocity and skin friction.

Nomenclature

Symbol	Entities	Dimension
a	: Outer radius of the cylinder	[L]
b	: Thickness of the cylinder	[L]
B_0	: Applied magnetic field	$[ML^2T^{-1}Q^{-1}]$
C_{fx}	: Skin friction coefficient	[---]
c_p	: Specific heat	$[L^2\theta^{-1}T^{-2}]$
f	: Dimensionless stream function	[---]
g	: Acceleration due to gravity	$[LT^{-2}]$
M	: Magnetic parameter	[---]
Nu_x	: Local Nusselt number	[---]
p	: Conjugate conduction resistant parameter	[---]
Pr	: Prandtl number	[---]
Q	: Heat generation parameter	[---]
T_f	: Temperature at the boundary layer region	[θ]
T_s	: Temperature of the solid of the cylinder	[θ]
T_b	: Temperature of the inner cylinder	[θ]
T_∞	: Temperature of the ambient fluid	[θ]
$\bar{u},\ \bar{v}$	: Velocity components	$[LT^{-1}]$
u, v	: Dimensionless velocity components	[---]
$\bar{x},\ \bar{y}$	: Cartesian coordinates	[L]
x, y	: Dimensionless cartesian coordinates	[---]
Greek Symbols	**Entities**	**Dimension**
β	: Co-efficient of thermal expansion	$[\theta^{-1}]$
ψ	: Dimensionless stream function	[---]
ρ	: Density of the fluid inside the boundary layer	$[ML^{-3}]$
ν	: Kinematic viscosity	$[L^2T^{-1}]$
μ	: Viscosity of the fluid	$[ML^{-1}T^{-1}]$
θ	: Dimensionless temperature	[---]
σ	: Electrical conductivity	$[MLT^{-3}\ \theta^{-1}]$
K_f	: Thermal conductivity of the ambient fluid	$[MLT^{-3}\ \theta^{-1}]$
K_s	: Thermal conductivity of the ambient solid	$[MLT^{-3}\ \theta^{-1}]$

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