Measuring the Factors of Examination Performance– a Stochastic frontier Production function: Case study on Jahangirnagar University

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Abstract

From an economic point of view, education can be regarded as a production process in which variety of study inputs are used to produce output. But the relationship between student study time allocation and examination performance is little understood. We model the allocation of student time into formal study (lectures & classes), self-study, private tuition, reading newspapers, watching TV, in mobile phone, leisure and sleeping and its relationship to university examination scores using a stochastic frontier production function. This case study uses detailed personal records from the students of Jahangirnagar University. The results suggest that within the formal system of teaching, time spent in both formal study and self-study are significant determinants of examination scores but the former may be up to six (6) times more important than the latter. We also find that time spent in private tuition, mobile phone and leisure are also significant determinants but time spent in private tuition has a very small effect and in mobile phone and leisure have negative effects on the performances of the students. So, the University authority should encourage the students to attend in the formal classes and lectures or even could make it compulsory. We also propose a modified stochastic frontier production function along with the test procedures when some of the parameters are constraints.

Key words: *Formal study, self-study, significant, efficiency.*
*JEL **I21; J2***

Introduction

There is a vast literature that attests to the importance of human capital, but a few of them have examined the mechanism of human capital acquisition, that is, exactly how do people acquire knowledge and what is the relationship between the learning environment and the educational achievement. From an economic point of view, education can be regarded as a production process, in which a variety of individual study inputs are used to determine a multidimensional output. Although there have been many studies of educational production, the evidence would suggest that we are still a long way from understanding how education is produced in terms of how study hours are transformed into knowledge. Most research, which models on how students achieve their examination grades, do not consider how students spend their time in the study processes.

The accepted technique for modeling the educational process of examination performance is the educational production function. This study models the existence of a university production

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function based on the individual student data. Among different types of production functions, a stochastic production frontier approach could be adopted, in which the disturbance term has two components, one to catch the usual statistical noise and the other to account for the existence of technical inefficiency. The data we use in our study comes from a case study from the students of Jahangirnagar University.

Objectives of the Study

The main objectives of study are as follows:

1) Modeling the effective use of student time by stochastic frontier production function.
2) To estimate as well as to test the parameters of the model.
3) To test if there is actually any technical inefficiency effect in the model, which we have used.
4) To propose a newly modified stochastic frontier production function where some of the parameters are restricted.

The Jahangirnagar University student time-survey

The data is taken from a survey in July 2007 on the students of Jahangirnagar University of the sessions 2000-2001, 2001-2002 and 2002-2003. Total number of students is 3277. In this survey, detailed information is collected relating to the amount of time the students dedicate to their normal activities. It is to be noted that the survey is not for official University administration but rather for only the research purposes. But the data can be used by the University administration for academic, teaching or assessment purposes. We have excluded any respondent with missing data from our analysis and those with more than 24 hours or 1440 minutes per day. But we have included any respondent between 23 hours or 1380 minutes and 24 hours or 1440 minutes per day because a major difficulty in any study of time use is to get respondents to accurately remember their time allocation. That is why, there is a time difference between 1440 minutes and the distribution of time use data. Our data have suggested that on an average the students allocate their daily time according to the following table:

Variables	Average time per day (min)	Variables	Average time per day (min)
Formal study	238.8	Watching TV	40.4
Self study	153.6	Mobile phone	7.4
Private tuition	145.4	Leisure	347.4
Reading newspaper	30.3	Sleeping	435.2

Juster and Stafford (1986) report that there are many potential biases in asking people to record time use. They suggest that asking respondents to keep a diary is a preferred survey method.[1] Unfortunately; this was not possible in this study. Juster and Stafford (1991) do however offer some reassurance to this study in an important respect. Namely, they suggest that reporting error is minimized when responses involve recording daily work patterns with regular schedules.[2] This finding is of most importance if we consider recording information about student study time. All students know how many hours of contact time are involved in their weekly timetable. Hence to calculate actual contact time, they only have to make some adjustment for non-attendance. Likewise the remainder of their weekly schedule will have a regular pattern, which may facilitate a reasonable estimate of self-study time.

Sample size determination through the approach based on precision rate and confidence level

To begin with, it can be stated that whenever a sample study is made, there arises some sampling errors, which can be controlled by selecting a sample of adequate size. Researcher will have to specify the degree of precision (d) that he wants in respect of his estimates concerning the population parameters. The degree of precision (d) is specified in such a way that the researcher may like to estimate the parameter within $\pm d$ of the true parameter with $\alpha\,\%$ level of significance. Keeping this in view, we can now explain the determination of sample size for our problem so that specified precision is ensured. The confidence intervals for the parameters (β) of our model is given bellow:

$$\hat{\beta} \pm Z_{\frac{\alpha}{2}}\, SE\left(\hat{\beta}\right).$$

If the difference between β and $\hat{\beta}$ or the acceptable error is kept within $\pm d$ with $(1-\alpha)\,\%$ confidence interval, then we can express the acceptable error d as follows:

$$d = Z_{\frac{\alpha}{2}} SE\left(\hat{\beta}\right) \quad => \; d^2 = Z^2_{\frac{\alpha}{2}} \operatorname{var}\left(\hat{\beta}\right) \quad => \; d^2 = Z^2_{\frac{\alpha}{2}}\, \hat{\sigma}^2 \left(diag\left(X'X\right)^{-1}\right) = Z^2_{\frac{\alpha}{2}} \frac{RSS}{n-k} \left(diag\left(X'X\right)^{-1}\right)$$

$$=> d^2 = Z^2_{\frac{\alpha}{2}} \frac{f(x)}{n-k} \; => n_0 = Z^2_{\frac{\alpha}{2}} \frac{f(x)}{d^2} + k \left[where,\; f(x) = RSS \left(diag\left(X'X\right)^{-1}\right) \; and \; n = n_0 \; for \text{ inf} inite \; population \right]$$

$$=> n = \frac{n_0}{1+\frac{n_0}{N}} \left(for\; finite\; population, that\; is\; N\; is\; known. In\; this\; case,\; d = Z_{\frac{\alpha}{2}}\, SE\left(\hat{\beta}\right) \sqrt{\frac{N-n}{N-1}} \right) \qquad (1)$$

where ,

X is a k-row vector, whose first element is 1 and the remaining elements are the logarithms of the k-1 input quantities ,
k = number of parameters to be estimated .

From the above, we have chosen that sample size, which is obtained by using the variance of largest coefficient. By using the formula in equation (1) *, the sample size is obtained as 300.*

Here, the coefficients are obtained by the ordinary least squares method using the data**.** These data are the **logarithms** of each of the **adjusted variables**, which are collected by a **pilot survey** of 50 observations each.

Parameters	Coefficients	Standard Errors	Variances	Diagonal of $\left(X'X\right)^{-1}$
Intercept	0.6187	2.4303	5.9066	541.2989
Formal study	0.6919	0.143	0.0204	1.8735
Self study	-0.0053	0.1782	0.0317	2.9093
Private tuition	0.014	0.0117	0.0001	0.0125
Reading newspaper	0.0022	0.0362	0.0013	0.1201
Watching TV	-0.0131	0.035	0.0012	0.1122
Mobile phone	-0.0468	0.0467	0.0022	0.2
Leisure	-0.134	0.1274	0.0162	1.4883
Sleeping	0.0799	0.1255	0.0158	1.4436

Multiple R	0.6736
R Square	0.4537
Adjusted R Square	0.3471
Standard Error	0.1045
Observations	50

	Degrees of freedom	Sum of squires	Mean sum of squares
Regression	8	0.3716	0.0464
Residual	41	0.4474	0.0109
Total	49	0.819	

$$\left(X'X\right)^{-1} = \begin{pmatrix} 541.2989 & -20.0243 & -35.2588 & 0.8228 & -1.5328 & -2.8674 & 1.4754 & -20.3498 & -21.036 \\ -20.0243 & 1.8735 & 0.7288 & 0.0076 & 0.0126 & 0.0494 & -0.0996 & 0.6098 & 0.4156 \\ -35.2588 & 0.7288 & 2.9093 & -0.113 & 0.0736 & 0.1774 & -0.1473 & 1.2586 & 1.5254 \\ 0.8228 & 0.0076 & -0.113 & 0.0125 & -0.002 & -0.0035 & 0.011 & -0.0212 & -0.0381 \\ -1.5328 & 0.0126 & 0.0736 & -0.002 & 0.1201 & 0.0219 & 0.0148 & 0.0146 & 0.0842 \\ -2.8674 & 0.0494 & 0.1774 & -0.0035 & 0.0219 & 0.1122 & 0.0434 & 0.0624 & 0.1293 \\ 1.4754 & -0.0996 & -0.1473 & 0.011 & 0.0148 & 0.0434 & 0.2 & -0.0746 & -0.0671 \\ -20.3498 & 0.6098 & 1.2586 & -0.0212 & 0.0146 & 0.0624 & -0.0746 & 1.4883 & 0.3306 \\ -21.036 & 0.4156 & 1.5254 & -0.0381 & 0.0842 & 0.1293 & -0.0671 & 0.3306 & 1.4436 \end{pmatrix}$$

Here, the adjusted variables means, the values of the variables, which are zero, are replaced by 0.01 because we have taken the logarithms of each of the variables and the logarithms of zero is not countable. The coefficients along with other information from this data are given in the above tables. So, from the above, we see that the largest coefficient is 0.6919 and the diagonal of $\left(X'X\right)^{-1}$ related to the largest coefficient is 1.8735. Again, the residual sum of squares (RSS) is 0.4474. So, we have that

$$f(x) = RSS \times diagonal\ \ of\ \left(X'X\right)^{-1}$$
$$= 0.4474 \times 1.8735 = 0.8382$$

Again, we have from the above that

$$n_0 = Z^2_{\frac{\alpha}{2}} \times \frac{f(x)}{d^2} + k = Z^2_{\frac{\alpha}{2}} \times \frac{RSS \times diagonal \;\; of \;\; \left(X^{/}X\right)^{-1}}{d^2} + k$$

$$\Rightarrow n_0 = (1.96)^2 \times \frac{0.8382}{(0.1)^2} + 9 = 331 \quad \left(Letting\,,\; d = 0.1\,,\; \alpha = 0.05 \right)$$

So, the final sample size is given by:

$$n = \frac{n_0}{1 + \frac{n_0}{N}} = \frac{331}{1 + \frac{331}{3277}} = 300 \quad \left(Since, \quad N = 3277 \right)$$

The Stochastic Frontier Production Function and its estimation procedure

Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) (Battese & Coelli, Springer, 2005) independently proposed the stochastic frontier[11] production function, which is outlined in the following:

$$\ln(y_i) = x_i\,\beta + v_i - u_i \quad ; \quad i = 1, 2, \ldots, n. \qquad (2)$$

where ,

$\ln(y_i)$ = The logarithm of the (scaler) output for the i^{th} firm,

x_i = k-row vector, whose first element is 1 and the remaining elements are the logarithms of the (k-1) input quantities used by the i^{th} firm,

$\beta = (\beta_1 \; \beta_2 \; \ldots \; \beta_k)^{/}$ = k-column vector of unknown parameters to be estimated,

u_i = a non-negative random variable, associated with technical inefficiency in production . of firms in the industry involved.

The random variable (v_i) accounts for measurement error and other random factors such as the effects of weather, strikes, luck, etc., on the value of the output variable together with the combined effects of unspecified input variables in the production function. Aigner, Lovell and Schmidt (1977) assumed that the v_i's were independent and identically distributed (i.i.d) normal random variables with mean zero and constant variance σ_v^2, independent of the u_i's, which were assumed to be (i.i.d) exponential or half-normal random variables. The stochastic frontier model is not, however, without problems. The main criticism is that there is generally no a priori justification for the selection of any particular distributional form for the u_i's. The specifications of more general distributional forms such as the truncated-normal (Stevenson 1980) and the two-parameter gamma (Greene 1990), have partially alleviated this problem, but the resulting efficiency measures may still be sensitive to distributional assumptions. The parameters of the stochastic frontier production function, defined by equation (2), can be estimated using the maximum-likelihood (ML) method, which requires numerical maximization of the following log-likelihood function:

$$\ln(L) = -\frac{n}{2}\ln\left(\frac{\pi}{2}\right) - \frac{n}{2}\ln\left(\sigma_s^2\right) + \sum_{i=1}^{n}\ln\left(1-\Phi\left(z_i\right)\right) - \frac{1}{2\sigma_s^2}\sum_{i=1}^{n}\left(\ln\left(y_i\right) - x_i\beta\right)^2. \qquad (3)$$

where ,

$$\sigma_s^2 = \sigma_u^2 + \sigma_v^2 \quad , \quad \gamma = \frac{\sigma_u^2}{\sigma_s^2} \quad , \quad z_i = \frac{\ln\left(y_i\right) - x_i\beta}{\sigma_s}\sqrt{\frac{\gamma}{1-\gamma}} \quad \text{and}$$

$\Phi(.)$ = distribution function of the standard normal random variable.

The maximum likelihood (ML) estimates of β , σ_s^2 and γ are obtained by finding the maximum of the above log-likelihood function. The maximum likelihood (ML) estimators are consistent and asymptotically efficient (Aigner, Lovell and Schmidt (1977), p.28). The computer program, **frontier version 4.1**, can be used to obtain the maximum likelihood (ML) estimates for the parameters of this model. This program uses a three-step estimation procedure, which is given bellow:

1) The first step involves calculation of the ordinary least squares (OLS) estimators of β and σ_s^2. Theses are unbiased estimators of the parameters in equation (1), with the exception of the intercept β_0 and σ_s^2.
2) In the second step, the likelihood function is evaluated for a number of values of γ between zero and one. In these calculations, the OLS estimates of σ_s^2 and β_0 are adjusted by:

$$\sigma_s^2 = \sigma_{OLS}^2\,\frac{\pi\left(T - K\right)}{T\left(\pi - 2\hat{\gamma}\right)} \quad \text{and} \quad \beta_0 = \beta_{0\,(OLS)} + \sqrt{\frac{2\,\hat{\gamma}\,\hat{\sigma}_s^2}{\pi}}\text{, respectively.}$$

The OLS estimates are used for the remaining parameters in β.

3) The final step uses the best estimates (that is, those corresponding to the largest log-likelihood value) from the second step as starting values in a DFP iterative maximization routine, which obtains the maximum likelihood (ML) estimates when the likelihood function attains its global maximum.

Approximate standard errors of the maximum likelihood (ML) estimators are calculated by obtaining square roots of the diagonal elements of the direction matrix from the final iteration of the DFP routine. The direction matrix from the final iteration is usually a good approximation for the inverse of the Hessian of the log-likelihood function, unless the DFP routine terminates after only a few iterations.

Prediction of Firm-level Technical Efficiencies and mean technical efficiency

The technical efficiency of the i^{th} firm is defined by:

$$TE_i = \exp\left(-u_i\right).$$

This involves the technical inefficiency effect $\left(u_i\right)$, which is unobservable. Even if the true value of the parameter vector β in the stochastic frontier model is known, the only difference, $e_i \equiv v_i - u_i$, could be observed. Battese and Coelli (1988) point out that the predictor of $\exp\left(-u_i\right)$ is obtained by using the following:

$$E\left(\exp\left(-u_i\right) \middle| e_i \right) = \frac{1 - \Phi\left(\sigma_A + \dfrac{\gamma e_i}{\sigma_A} \right)}{1 - \Phi\left(\dfrac{\gamma e_i}{\sigma_A} \right)} \exp\left(\gamma e_i + \frac{\sigma_A^2}{2} \right). \qquad (4)$$

The technical efficiency predictor implemented in the **FRONTIER** computer program is obtained by replacing the unknown parameters in equation (4) with their maximum likelihood estimators. Because the individual technical efficiencies of sample firms can be predicted, an estimator for the mean technical efficiency is the arithmetic average of the predictors for the individual technical efficiencies of the sample firms. This is what is calculated by **frontier version 4.1**.

Tests of Hypotheses

For the frontier model, defined by equation (2), the null hypothesis that there are no technical inefficiency effects in the model, can be conducted by testing the null and alternative hypotheses as follows:

$$H_0 : \gamma = \frac{\sigma_u^2}{\sigma_s^2} = 0 \quad \text{vs} \quad H_A : \gamma = \frac{\sigma_u^2}{\sigma_s^2} > 0$$

The above hypothesis can be tested by one-sided generalized likelihood ratio test. The test statistic is given by:

$$LR = -2 \ln\left(\frac{L\left(H_0\right)}{L\left(H_A\right)} \right) = -2 \left[\ln\left(L\left(H_0\right) \right) - \ln\left(L\left(H_A\right) \right) \right] \qquad (5)$$

where,

$L\left(H_0\right)$ and $L\left(H_A\right)$ are the values of the likelihood function under the null and alternative hypotheses, respectively.

Under the null hypothesis, this test statistic is assumed to be asymptotically distributed as a mixture of chi-square distribution with degrees of freedom equal to the number of restrictions involved (in this instance one), namely, $\frac{1}{2}\chi_0^2 + \frac{1}{2}\chi_1^2$. At $\alpha\,\%$ level of significance, the critical value is $\chi_1^2\ (2\alpha)$. It is to be noted here that the regular (two-sided) generalized likelihood ratio test was included in the Monte-Carlo experiment in Coelli (1995) and shown to have incorrect size (too small), as expected.

The proposed modified stochastic frontier production function and its estimation procedure

We have discussed earlier about the estimation procedures of the parameters of the stochastic frontier model with the computer program **frontier version 4.**1, in which there have been no constraints about the parameters. That is, the **frontier version 4.1** is unable to estimate the parameters of the model when some of the parameters are constraints.

But in practical situation, we deal with most of the econometric problems in which the sign of the parameters are known in advance. Estimation and testing procedures of such a stochastic frontier production function model with respect to some constraints are quite different from the usual stochastic frontier production function. So, we propose a modified stochastic frontier production function for our data in which some of the parameters are constraints. The newly proposed modified stochastic frontier production function for our data can be written as follows:

$$\ln(y_i) = x_i\,\beta + v_i - u_i \quad ; \quad i = 1, 2, \ldots, n. \tag{6}$$

subject to the constraints $\beta \in C$.

where, $\beta \in R^p$ is a sub-vector of an unknown parameter space $\Theta \in R^s$ and C is a subset of R^p. The parameters of the proposed modified stochastic frontier production function, defined by equation (6), can be estimated using the maximum-likelihood (ML) method, which requires numerical maximization of the following likelihood function:

$$\ln(L) = -\frac{n}{2}\ln\left(\frac{\pi}{2}\right) - \frac{n}{2}\ln\left(\sigma_s^2\right) + \sum_{i=1}^{n}\ln\left(1-\Phi\left(z_i\right)\right) - \frac{1}{2\sigma_s^2}\sum_{i=1}^{n}\left(\ln\left(y_i\right) - x_i\,\beta\right)^2. \tag{7}$$

subject to the constraints $\beta \in C$.

where, $\beta \in R^p$ is a sub-vector of an unknown parameter space $\Theta \in R^s$ and C is a subset of R^p. As it is noted earlier that the **frontier version 4.1** is unable to estimate the parameters of this type of model, we have to write some programs in **Gauss** or **SAS** or in other suitable languages to obtain the maximum likelihood estimators of β, σ_S^2 and γ. After obtaining the maximum likelihood estimators of this modified stochastic frontier production function, we can estimate the individual technical efficiencies and the mean technical efficiency as in the same way we have estimated these for the stochastic frontier production function without any constraint. We can also test whether there is any technical inefficiency effect is present in this modified model or not as in the same way we have done in the usual stochastic frontier production function. But the testing procedure is quite different for testing the coefficients of the variables in case of modified stochastic frontier production function. In this case, the test statistic follows the weighted mixture of chi-squire distribution.

Econometric results

In this section, we discuss the results obtained using the **frontier version 4.1**. The results of the finales mates are as follows:

The final mle estimates are:

	Coeff.	t-ratio
Intercept	-1.257	-1.87
Formal study	0.7879	18.92
Self-study	0.1221	2.45
Private	0.0139	4.24
News paper	0.0084	0.78
Watching TV	0.0107	1.11
Mobile	-0.0562	-3.78
Leisure	-0.1517	-3.58
Sleeping	0.2168	5.48
Sigma-squared	0.0203	8.59
Gamma	0.9144	29.35

log likelihood function = 303.77
LR test of the one-sided error = 27.74
Mean technical efficiency = 0.9013

The interpretations of the results are as follows:

1) From the above results, it is seen that the calculated value of the likelihood ratio test of the one-sided error (27.74) is greater than the tabulated value (2.71). So, the null hypothesis of the absence of the technical inefficiency effect is rejected. That is, the use of the stochastic frontier production function is appropriate.
2) It is also seen that the coefficients of time spent in reading newspaper and watching TV are insignificant.
3) The coefficients of time spent in formal study, self study, private tuition, mobile phone, leisure and sleeping are significant.
4) Time spent in the formal lectures has a positive effect on the performance of the students in examination and the coefficient is $\mathbf{0.7879}$. That is, for one unit increase in the variable formal lectures on an average the score will increase by $\mathbf{0.7879}$ units, holding others constant. This clearly shows that the students who attend in formal lectures are those who are more capable of getting high score. Time spent in self-study has also a positive effect on the performance of the students in the examination and the coefficient is $\mathbf{0.1221}$, which is less than that of the formal lectures.

   These results suggest that within the formal system of teaching in Jahangirnagar University, both formal study and self-study are significant determinants of exam scores but that the former may be up to $\mathbf{6}$ times more important than the latter.

   Because each person has a finite capacity to take in subject matter. Hence, after a period of intensive self-study, a person's capacity of learning new concepts by further time input may be strictly constrained. Hence, the efficient allocation of time for self-study must be short. Another explanation of this result is that opportunity for self-study hours is strictly constrained (one has to allow for formal study time, leisure, sleep etc). So, most of the students have a limited range of hours to chose the time for formal study.

5) Time spent in private tuition has also a positive effect and the coefficient is $\mathbf{0.0139}$, which is very small. That is, for one unit increase in the variable in private tuition on an average the score will increase by $\mathbf{0.0139}$ units, holding others constant. Because most of the students of Jahangirnagar University spend only a few times in private tuition, which has very low effect in their examination performance.
6) Time spent in mobile phone has a negative effect and the coefficient is $\mathbf{-0.0562}$, which is very small. That is, for one unit increase in the variable in mobile phone on an average the score will decrease by $\mathbf{-0.0562}$ units, holding others constant. This implies that although the coefficient is very small, it has a negative effect on the performance of the students of Jahangirnagar University.
7) Time spent in leisure has also a negative effect and the coefficient is $\mathbf{-0.1517}$. That is, for one unit increase in the variable in leisure on an average the score will decrease by $\mathbf{-0.1517}$ units, holding others constant. This clearly shows that the students who spend their maximum time in leisure are those who are less capable of getting high score in the examination.
8) Time spent in sleeping has a positive effect and the coefficient is $\mathbf{0.2168}$. That is, for one unit increase in the variable in sleeping on an average the score will increase by $\mathbf{0.2168}$ units, holding others constant. This means that the students who spend some of the times for sleeping are capable of getting good scores, because without sleeping no one is able to gat high score in the examination.

Summary and conclusions

From the above results and interpretations, we provide some suggestions to the students as well as to the University authority, which are as follows:

- The students have to spend maximum of their times in formal lectures and classes than in self-studies and should not spend their maximum of the times in mobile phone and in leisure.
- They should sleep every day for some period of time. This will help them to do all others works properly.
- They have to use their mobile phone only in crying need, excessive use of this will a effect their performance in the examination.
- The University authority should encourage the teachers to take the classes seriously and also the students to attend the lectures and classes, seminars, symposiums, by providing logistic supports etc. or even could make it compulsory.
- The government could take necessary steps to encourage the students to attend the lectures and classes regularly through the University Grant Commission and other organizations by seminars, symposiums, some extra-curriculum activities, some logistic supports, training of the teachers as well as the students etc.

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Endnote

[1] The model is called a frontier production function because here the technical inefficiency effect is separated from the usual statistical noise and thus optimum output is obtained from a given set of minimum inputs.